Equivalisation (once again)[1]

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Income inequality and poverty statistics are often adjusted for differences in the size and composition of households, with common practice being to use a set of "equivalence scales": a function of the number of adults and children in the household. However, different – largely ad-hoc – scales have been adopted by international organisations and national statistical organisations with little explicit justification. We derive equivalence scales from the expenditure patterns of households using data from the 1987-2015 Irish Household Budget Survey. We find that the Engel, Rothbarth and demand system approaches all yield scales for children that are substantially smaller than those used by ad-hoc official scales, with estimates from AIDS and QUAIDS demand systems appearing to decline over time. Such a decline is particularly noticeable when a 3SLS estimator is used to correct for the potential endogeneity of total expenditure, which also leads to substantially smaller estimated scales for both adults and children. This suggests an increase in economies of scale over the period we examine, with important implications for the measurement of poverty and inequality.

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1. Introduction

Equivalisation is a crucial, if often overlooked, part of calculating statistics on household income inequality and poverty. This process adjusts ("equivalises") the income of households of differing sizes and composition, with common practice being to divide the income of a household by a set of "equivalence scales": a function of the number of adults and children in the household. As well as reflecting – infrequently discussed – implicit value judgements (Deaton and Muellbauer, 1986; Atkinson et al., 1995), research has shown that the choice of these scales can make meaningful differences to measures of income inequality and poverty (e.g. Regan and Kakoulidou 2025; Mysikova et al., 2022; Goedemé et al, 2019; Aaberge and Melby, 1998; Jenkins and Cowell, 1994).

Despite this, different – largely ad-hoc – equivalence scales have been adopted by international organisations with little explicit justification. For example, Eurostat use what is known as the "modified OECD scale" in the construction of their statistics on income distributions. This assigns a weight of 1 to the first adult, 0.5 to each additional adult aged 14 or older, and 0.3 to each child under 14. The OECD, by contrast (and somewhat counterintuitively), use the square-root of household size for their income distribution statistics database.

In addition, many countries have also adopted – again, largely ad-hoc – country-specific equivalence scales.[2] For example, the Government of Ireland (1997; 2020) and Irish Central Statistics Office (CSO, 2022) use a 'national scale' that assigns a weight of 1 to the first adult in a household, 0.66 to each additional adult aged 14 or older and 0.33 to each child under 14.[3]

Yet there has been relatively limited effort to assess which – if any – of these scales should be adopted when calculating statistics on income inequality and poverty, particularly over a longer horizon. This paper does so using three different approaches to derive equivalence scales from the patterns of expenditure observed in the Irish Household Budget Surveys (HBS) covering the period 1987-2015.

The first approach is based on the assumption that households of differing composition have the same standard of living if they spend the *same share* of their total expenditure on some necessary good e.g. food (Engel, 1895). The second assumes such households have the same standard of living if they spend the *same amount* on some set of "adult" goods e.g. adult clothing, alcohol etc. (Rothbarth, 1943).[4]

However, among the shortcomings of these approaches is that they are sensitive to the precise choice of necessary or adult good, with the appropriate choice of good not evident *a priori* (Deaton et al., 1989; Lewbel and Pendakur, 2008). To address this concern, we also adopt a third approach which involves estimating the parameters of the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980a) and the Quadratic Almost Ideal Demand System (QUAIDS) of Banks et al. (1997). Using these demand system models allow us to derive equivalence scales on the basis of households' expenditure patterns across all goods in a way that is more consistent with consumer theory.

We find that estimates from all three approaches yield equivalence scales for children that are smaller than the ad-hoc scales typically used. Across all methods, the implied scale for children is 0.10-0.30 compared to the modified OECD or Irish national scale of 0.30 and 0.33 respectively. Our estimates of

[2] Among EU countries, the national statistical organisations of Denmark, Italy, Ireland, Romania, Slovakia and Sweden all use a country-specific equivalence scale.

[3] The rationale for this national scale – one of three used by Callan and Nolan (1987) in early work on the distribution of income in Ireland – is based on the relativities of social welfare payments in the mid-1980s.

[4] Such goods must be consumed only by adults: as a result, the Rothbarth method can only be used to derive an equivalence scale for children and not additional adults in a household.

child equivalence scales derived from (QU)AIDS models are particularly small, especially in recent waves of data when they are below 0.10.

Although estimates from such demand systems are more consistent with consumer theory, they remain subject – as with Engel and Rothbarth approaches – to concerns about endogeneity. Foremost among these is the potential endogeneity of total expenditure which is plausibly correlated with the error term because of unobserved preferences and measurement error (Banks et al., 1997).

To account for this, we follow Browning (1994) – among others – in instrumenting total expenditure with total income through a 3 stage least squares (3SLS) estimator. While our potential endogenous Engel, Rothbarth and (QU)AIDS estimates generally yield adult scales of 0.50-0.60, instrumenting total expenditure to account for endogeneity gives (QU)AIDS estimates that are far lower, particularly in more recent waves of data.

We then examine how measures of income inequality and poverty in Ireland using all the estimated scales vary. While the downward trend in income inequality (as measured by the Gini coefficient) does not appear to be sensitive to the choice of scale, rates of income poverty – particularly child poverty – are. Although most of the scales yield an overall income poverty rate of c.15% in 2015, the income poverty rate for children in that year varies between 8.8% using the scales derived from the AIDS model to 17.7% using the modified OECD equivalence scale. The change in child poverty over time also varies substantially across the scales, illustrating the importance of the choice of equivalence scale in the measurement of poverty.

Our contribution to the literature is threefold. Firstly, we build on earlier work that sought to derive equivalence scales from patterns of household expenditure (e.g. Conniffe and Keogh, 1988; Conniffe, 1992; Blundell and Lewbel, 1991; Banks and Johnson, 1994; Bargain et al., 2010; Garvey et al., 2011). Such work has typically adopted a single approach and focused on a single demographic group (e.g. children). We use a range of approaches to derive equivalence scales for both adults and children, in addition to showing the importance of addressing the endogeneity of total expenditure in demand system based approaches.

In addition, by exploiting the near three-decade horizon covered by our data we show that the equivalence scales implied by expenditure patterns do not appear to be constant over time.[5] This comparison of estimated equivalence scales over time and their effect on estimates of poverty and inequality is largely absent from the international literature despite the obvious concern that household economies of scale may change over time.

Finally, we contribute to the large literature that has shown measures of income inequality and poverty are sensitive to the choice of scale (e.g. European Commission, 2022; Regan and Kakoulidou 2025; Mysikova and Zelinsky, 2019; Aaberge and Melby, 2005; Figini, 1998; Jenkins and Cowell, 1994; Banks et al., 1994; Coulter et al., 1992). In particular, we show that estimates of child poverty are considerably higher using official ad-hoc scales compared to our estimated scales, and can be sensitive to the use of contemporaneous (changing) rather than fixed scales as common in the literature.

The rest of the paper proceeds as follows. Section 2 outlines our empirical approach and data. Section 3 presents our estimates of equivalence scales using the Engel, Rothbarth and demand system approaches. Section 4 explores how these estimates translate into measures of income inequality and poverty while Section 5 concludes.

[5] Daley *et al*. (2020) find a similar result but using just the Engel approach.

2. Empirical Approach and Data

Three broad classes of equivalence scale have been adopted in research on the distribution of income (Atkinson et al, 1995). "Expert" scales – such as the modified OECD and Irish national equivalence scales – are based on the judgement of policy makers and academics. One criticism of these scales is that they are not necessarily grounded in economic theory or empirical data and could be considered ad-hoc in nature. Subjective survey scales are instead based on respondents self-assessed levels of income adequacy (e.g., Kapetyn and van Praag, 1978), while consumption or expenditure scales – which are the focus of this paper – are based on patterns of household expenditure. This section outlines our empirical approach to estimating such scales, the data we use and the estimates.

2.1 Empirical approach

The first approach we use is rooted in Engel's law, the empirical observation that the proportion of a household's expenditure that goes on food (the "food share") is a *decreasing* function of income and total expenditure. Accordingly, the Engel curve for food (which traces out the food share as a function of income or total expenditure) should slope downwards as one moves from lower to higher levels of total expenditure.[6] If household welfare is increasing in total expenditure, it follows that there is a monotonically decreasing relationship between the food share and household welfare; and, in particular, that two households have the same level of welfare if their food shares are equal.

The Engel curve can be parametrically specified as a function of total expenditure. In our analysis, we focus on the widely used Working-Leser form which models the food share as a linear function of the log of income (Deaton, 1981; Garvey et al, 2011).

Denoting by w_i the expenditure share of the i^{th} good, the Working-Leser form is

$$w_i = \beta_0 + \beta_1 lnY + \beta_2 n_c + \beta_3 n_a + \varepsilon_i \quad (1)$$

where Y is total household expenditure, and n_c and n_a are the number of children and adults in the household respectively. In our analysis, we consider three sets of household expenditure: food, clothing and housing costs, as a well the combined expenditure on these three goods. Although Engel's law relates to food, clothing and housing costs can also be considered as necessary goods, with research often using them in conjunction with food to estimate equivalence scales (Garvey et. al., 2011).

The parameters of equation 1 are estimated by OLS. Then, let n_a^0 be the number of adults in the reference household, n_c^0 the number of children in the reference household, and $n^0 = n_a^0 + n_c^0$ the size of the reference household (a common choice being, e.g., $n^0 = n_a^0 = 2$, $n_c^0 = 0$). For a "comparison household" type with number of adults n_a^h, number of children n_c^h, and total size n^h, the scale which converts a level of income for the reference household to an equivalent level of income for the comparison household is (Deaton and Muellbauer, 1986):

$$\frac{n^h}{n^0} \cdot exp\left[\frac{\beta_3}{\beta_1}\left(n_a^h - n_a^0\right) + \frac{\beta_2}{\beta_1}\left(n_c^h - n_c^0\right)\right] \quad (2)$$

This formula yields constant equivalence scales which do not depend on the level of total expenditure. Given the non-linear dependence of the scale on parameters, we follow Phipps and Garner (1991) in using the delta method (i.e., a Taylor series expansion around the point estimates) to compute standard errors for scales.

[6] For more details regarding the Engel model see (Deaton and Muellbauer, 1980b, ch.8).

Modern approaches to Engel curve estimation have found nonlinear Engel curves for a number of commodities, such as alcohol and clothing. This has prompted the use of non-parametric estimation methods which make smoothness or differentiability requirements of the Engel curve but do not require the specification of a particular functional form (Engel and Kneip, 1996; Banks, Blundell and Lewbell; 1997). We supplement our parametric estimation with estimates from kernel regressions, following Blundell et al (1997) in using the Epanechnikov kernel for the continuous variable lnY and the Liracine kernel for the discrete variables n_a and n_c.

The Engel method has been subject to the criticism that it tends to overestimate the true equivalence scales for children (Nicholson, 1976; Deaton and Muellbauer, 1986). The Rothbarth method aims to overcome this objection.[7] Like the Engel method, it posits a single good the consumption of which is argued to correspond to levels of household welfare. However, the model does not rest on an empirical regularity like Engel's Law. Instead, it is stipulated *a priori* that some "adult-only" good is consumed only by parents and not by children. Further, parents' preferences for this good are assumed to be invariant under changes to the number of children: adding children to the household has only an income effect on parents' consumption of this good, not a substitution effect (the so-called "demographic separability" of parental and child preferences described by Deaton *et. al.*, 1989).

Under these assumptions, the Rothbarth model takes the absolute amount of expenditure on the "adult-only" good as an indicator of parental welfare, rather than its budget share (Tsakoglou, 1991; Banks and Johnson, 1994). On the Rothbarth model, different households spending the same amount on this "adult-only" good are at the same level of welfare. The assumptions of the model serve only to identify equivalence scales for children: the model says nothing about the effect on preferences for or consumption of the adult good when an additional adult enters the household. Accordingly, the Rothbarth method only yields estimates of child equivalence scales.

Formally, we choose a level of (log) total expenditure Y at which to evaluate the scales, Y^0, and set the variables n_a^0, n_c^0 to their reference values. We then solve for the equivalent level of income Y^h for a comparison household with characteristics n_a^h, n_c^h as:

$$Y^h = exp\left(\frac{Y^0 - \beta_0 - \beta_2 n_c^h - \beta_3 n_a^h}{\beta_1}\right) \quad (3)$$

The equivalence scale itself is calculated as is Y^h/Y^0 and is reported at the sample median or mean level of expenditure (Tsakloglou, 1991; Balisacan, 1991; White and Masset, 2002; Bargain and Donni, 2011).

Though the Rothbarth method can avoid the tendency of the Engel method to overstate child equivalence scales, it has several shortcomings of its own. In particular, the identification assumption – that some goods are consumed only by parents and that parents' preferences for those goods are unchanged with the arrival of children to the household – is questionable. The Rothbarth model also makes no allowance for differences in *inter-household* preferences for the adult good, which is particularly relevant for goods such as alcohol, tobacco, and gambling. Perhaps not unrelatedly, it has been observed that child scales estimated by the Rothbarth method are very sensitive to the choice of outcome good (Lancaster and Ray, 1997).

Furthermore, all such "single equation" methods are subject to the objections of allowing no role for *price variation* in goods to affect equivalence scales (e.g., the assumption of the models that the addition of children to a household has a pure income effect) and of having no rigorous grounding

[7] For formal treatments of the Rothbarth method, see Gronau (1988, 1991).

microeconomic theory (links to utility or welfare are established via empirical regularities or *a priori* stipulations). For these reasons, we also estimate equivalence scales using a demand-system approach. To formalise this approach, allow the household's *expenditure function* $E(\boldsymbol{p}, u)$ to depend on a vector of demographic variables $\boldsymbol{z}$. Then, the equivalence scale (S) can be written:[8]

$$S(\boldsymbol{p}, u, \boldsymbol{z}_0, \boldsymbol{z}_h) = \frac{E_h(\boldsymbol{p}, u, \boldsymbol{z}_h)}{E_0(\boldsymbol{p}, u, \boldsymbol{z}_0)}$$

where $\boldsymbol{z}_0$ is a vector of demographic characteristics for the reference household and $\boldsymbol{z}_h$ for another household.

With this approach, an equivalence scale can be obtained by choosing functional forms for the reference household's expenditure function E_0 and the other household's expenditure function E_h. Equivalently, using Shephard's Lemma, one can specify the household's demand function or budget share function for each commodity, along with demographic variables. [9]

Following the "generalised cost scaling" approach of Ray (1982, 1983),[10] we incorporate demographic variables into the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980a). In our commodity classification, we follow the specification of Savage (2016) who uses the same data as we do to explore the impacts of hypothetical indirect tax reforms.

The budget share equation for the i^{th} good of the standard AIDS is:

$$w_i = \alpha_i + \textstyle\sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln\left(\frac{y}{P}\right) \quad (4)$$

Where p_j is the price of the j^{th} good, y is total expenditure, Greek letters are parameters to be estimated, and P is a translog price index:

$$\ln P = \alpha_0 + \textstyle\sum_{i=1}^{n} \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln p_i \ln p_j \quad (5)$$

The parameter α_0 is specified by the researcher, with the common choice being to set it equal to or slightly less than the largest value of log total expenditure observed in the data (Deaton and Muellbauer, 1980b; Banks et al., 1997).[11]

Demographic variables are incorporated into the model following Ray (1983) by letting:

$$w_i = \alpha_i + \textstyle\sum_{j=1}^{n} \gamma_{ij} \ln p_j + \left(\beta_i + \boldsymbol{\eta}_i^T \boldsymbol{z}\right) \ln\left(\frac{y}{\overline{m}_0(\boldsymbol{z})P}\right) \quad (6)$$

where $\boldsymbol{z}$ is a vector of demographic variables and $\overline{m}_0(\boldsymbol{z}) = 1 + \boldsymbol{\rho}^T \boldsymbol{z}$; the ρ and η vectors are parameters to be estimated. In our model, demographic variables include the number of children in

[8] Incidentally, the definition of the "true" equivalence scale as a ratio of expenditure functions is almost identical to the definition of the "true" or Könus price index from index number theory.

[9] Shephard's Lemma asserts that the partial derivative of $E(\boldsymbol{p}, u)$ with respect to p_i is the (Hicksian or compensated) demand for good i; equivalently, that the partial derivative of $\ln E(\boldsymbol{p}, u)$ with respect to p_i is the budget share for good i.

[10] Pollak and Wales (1981) outline five other methods for incorporating demographic variables into demand system analysis.

[11] In keeping with the literature, we also impose some other parameter restrictions on the basis of theory, e.g. symmetry of the Hicks-Slutsky matrix and linear homogeneity of the demand functions. For details on how these restrictions are imposed see (Savage,2016, pg.375) or (Poi, 2012).

the household and the number of adults in the household.[12] The parameters ρ_1, ρ_2 are the equivalence scales for adults and children, and the $\boldsymbol{\eta}$ parameters are estimates of the sensitivity of the scales to the prices of our six commodities.

We also estimate equivalence scales using the quadratic almost-ideal demand system or QUAIDS model of Banks, Blundell, and Lewbel (1997), again employing the demographic scaling method of Ray (1983) to incorporate demographic variables. Without demographic variables, the budget share equation for the i^{th} good in a QUAIDS specification is:

$$w_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln\left(\frac{y}{P}\right) + \frac{\lambda_i}{b(P)} \left[\ln \frac{y}{P}\right]^2 (7)$$

where $b(P)$ is defined as:

$$b(P) = \prod_{i=1}^{n} p_i^{\beta_i}$$

If $\lambda_i = 0$ for $i = 1, \dots, n$, QUAIDS reduces to AIDS. Since the first three terms of the QUAIDS budget share equations are identical to the AIDS budget shares, demographic variables are incorporated into these terms in the QUAIDS budget share equations as in equation (6).

For the final term, the corresponding expression in the demographically-scaled QUAIDS is:

$$\frac{\lambda_i}{b(P)c(P,z)} \left[ln \frac{y}{\overline{m_0}(z)P}\right]^2$$

Where all terms are as defined before, and $c(P,z)$ is defined as:

$$c(P,z) = \prod_{i=1}^{n} p_i^{\eta_i{}^T z}$$

As in the non-demographic case, the demographically-scaled QUAIDS reduces to the demographically-scaled AIDS model if $\lambda_i = 0$ for $i = 1, \dots, n$.

While these QUAIDS and AIDS approaches are rooted in and more consistent with consumer theory, they remain subject to concerns about endogeneity. In particular, total expenditure – which appears on the right-hand side of equations 4 and 7 – is likely to be correlated with the error term. This might be because unobserved preferences for a particular category of expenditure (and so expenditure shares) are correlated with overall expenditure, or because of measurement error (Banks et al., 1997).

To account for this potential endogeneity, we present estimates from a three-stage least squares (3SLS) approach for our demographically-scaled AIDS and QUAIDS specifications, broadly following Lecoq and Robin (2015).[13] This involves running a first-stage regression for total expenditure y on prices p, demographic variables z, and an instrument: in our case, following Browning (1994), household income.

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[12] As in Michelini (2001, pg.386), we subtract one from the adult variable to obtain the appropriate reference household.

[13] The Stata command **`aidsills`** developed by Lecoq and Robin (2015) implements this 3SLS methodology with AIDS and QUAIDS models which do not incorporate demographic variables or do so via the demographic translation approach of Pollak and Wales (1981). To maintain consistency with the demographic scaling approach of Ray (1983), we instead implement the 3SLS approach with demographically-scaled AIDS and QUAIDS specifications using Stata's `nlsur` command.

Maintaining the $i = 1, \dots, n$ subscript for commodities and introducing a $h = 1, \dots, H$ superscript ranging over households, we can orthogonally decompose the error term in our AIDS and QUAIDS budget equations as:

$$u_i^h = \xi_i \hat{v}^h + \varepsilon_i^h \quad (8)$$

Where $\hat{v}^h$ is the residual from a first-stage regression for total expenditure y^h, $\xi_i, i = 1, \dots, n$ are parameters to be estimated, and ε_i^h is conditional-mean-independent of total expenditure. Including the term $\xi \hat{v}^h$ to our estimating equations (6) and (7) amounts to estimating the AIDS and QUAIDS systems by 3SLS; furthermore, one can directly test for exogeneity of total expenditure by testing the parameter restrictions $H_0: \xi_i = 0$ for $i = 1, \dots, n$.

Two other common specifications are found in the literature: *Buhmann-type* or *constant* elasticity scales, where the log of total expenditure is regressed on the log of household size (Buhmann *et al.*, 1988); and *two-parameter-type* scales, where family size is decomposed into the sum of adults and children and another parameter is added (Banks and Johnson 1994, Jenkins and Cowell 1994). To allow for comparison with earlier research, we provide estimated scales from these specifications in the Appendix, following the formulations given by Jenkins and Cowell (1994).

2.2 Data

We use data from all waves of the Irish Household Budget Survey (HBS) available between 1987 and 2015.[14] The survey follows a nationally representative sample of 6,000 to 7,000 households for two weeks and asks them to record their expenditures. It also collects rich demographic information such as age, sex and household size. Our data on prices are taken from the subindices of the Consumer Price Index published by the Central Statistics Office.[15]

To derive equivalence scales using the Engel method, we use budget shares for food, clothing and housing costs, as well as the combination of these three. To derive equivalence sales using the Rothbarth method, we must select some set of "adult" goods which are assumed to only be consumed by adults. Following the existing literature in this area, we use budget shares for adult clothing, alcohol and tobacco and gambling as well as the combination of the three. We also use the available data on household size and composition (number of adults and children) in the HBS. Following the definition of EUROSTAT, in our main results we treat all individuals younger than 14 years of age as dependent children with older individuals treated as adults.

3. Equivalence scale estimates

In this section, we present estimates of scales from our three approaches. Table 1 presents these estimates for adult scales over time with standard errors shown in parenthesis. Both the Engel and uninstrumented demand system approaches yield estimates that largely fall in the range 0.50-0.70: consistent with the values used by the modified OECD and the Irish National scales. A notable exception is our QUAIDS estimate for 2009, which is close to 0.

[14] These are the 1987, 1994-95, 1999-2000, 2004-05, 2009-10 and 2015-16 waves of the HBS.

[15] We use series taken from CPM01 and CPM03 available at https://data.cso.ie

Table 1: Estimated Equivalence Scales for Additional Adults

	Food (WL)	Food (K)	Combined (WL)	Combined (K)	AIDS	AIDS 3SLS	QUAIDS	QUAIDS 3SLS
1987	0.653 (0.003)	0.658 (0.003)	0.663 (0.005)	0.663 (0.003)	0.576 (0.031)	0.548 (0.036)	0.584 (0.030)	0.533 (0.031)
1994	0.670 (0.004)	0.607 (0.003)	0.664 (0.006)	0.654 (0.006)	0.602 (0.036)	0.611 (0.039)	0.604 (0.032)	0.585 (0.034)
1999	0.684 (0.004)	0.748 (0.006)	0.683 (0.006)	0.703 (0.012)	0.529 (0.029)	0.467 (0.037)	0.533 (0.030)	0.331 (0.025)
2004	0.685 (0.005)	0.684 (0.008)	0.668 (0.008)	0.661 (0.010)	0.567 (0.034)	0.395 (0.043)	0.604 (0.033)	0.361 (0.029)
2009	0.717 (0.008)	0.775 (0.023)	0.63 (0.009)	0.625 (0.017)	0.598 (0.048)	0.477 (0.059)	0.003 (0.017)	0.354 (0.039)
2015	0.701 (0.008)	0.712 (0.011)	0.664 (0.008)	0.649 (0.006)	0.513 (0.042)	0.115 (0.012)	0.471 (0.040)	0.180 (0.029)

Note: Authors' calculations using data from various editions of the Household Budget Survey. (K) indicates Kernel and (WL) Working-Leser form estimates. Standard errors in parenthesis.

However, our estimate of 0.354 for the same year from our 3SLS QUAIDS specification is very similar to those obtained from the same specification for 1999 and 2004. This specification instruments total expenditure with total income, and, apart from the year 2009, yields estimates that are generally much smaller than those from the uninstrumented QUAIDS specification. Likewise, estimates from our 3SLS AIDS specification are generally much smaller than those from the uninstrumented AIDS specification, particularly in more recent data years. Both 3SLS specifications also yield estimates of scales that are lower (<0.20) in more recent years and appear to decline over time, although not monotonically.

Table 2 presents our estimates of child scales, with standard errors again shown in parenthesis. As with the estimated adult scales, instrumenting likely endogenous total expenditure with income results in substantially lower estimates from our AIDS specification and a more plausible estimate for 2009 from our QUAIDS specification. However, the majority of our estimates – including those from the uninstrumented AIDS and QUAIDS specifications – are well below the values of 0.30 and 0.33 used by modified OECD and national scales respectively.

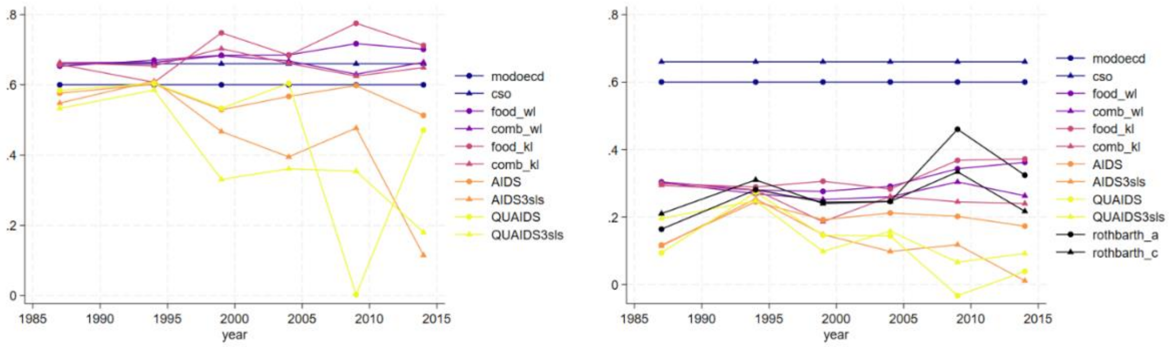
An exception to this general pattern is the estimated scales from the Engel method with food as an outcome good, which are always close to and often greater than 0.30. As discussed in Section 2, Deaton and Muellbauer (1986) – among others – have critiqued the Engel approach as likely to yield excessively large child scales and to overcompensate households for the presence of young children because most of the expenditure on young children is related to food. Consistent with this, we find that estimated child scales from the Engel method are notably smaller when a combined "bundle" of necessary goods is used as the outcome good as opposed to solely food.

Table 2: Estimated Equivalence Scales for Children

	Food (WL)	Food (K)	Combined (WL)	Combined (K)	AIDS	AIDS 3SLS	QUAIDS	QUAIDS 3SLS	Rothbarth (Alcohol)	Rothbarth (Combined)
1987	0.304 (0.006)	0.299 (0.006)	0.302 (0.008)	0.294 (0.007)	0.117 (0.021)	0.114 (0.037)	0.094 (0.020)	0.196 (0.031)	0.164 (0.034)	0.210 (0.026)
1994	0.280 (0.009)	0.289 (0.004)	0.269 (0.011)	0.280 (0.022)	0.245 (0.042)	0.254 (0.047)	0.273 (0.035)	0.248 (0.038)	0.281 (0.044)	0.310 (0.033)
1999	0.276 (0.008)	0.306 (0.016)	0.252 (0.012)	0.186 (0.016)	0.192 (0.025)	0.148 (0.048)	0.146 (0.027)	0.098 (0.026)	0.244 (0.050)	0.240 (0.036)
2004	0.291 (0.010)	0.283 (0.007)	0.260 (0.009)	0.261 (0.020)	0.212 (0.029)	0.098 (0.055)	0.144 (0.033)	0.158 (0.030)	0.246 (0.039)	0.246 (0.039)
2009	0.343 (0.015)	0.368 (0.031)	0.304 (0.017)	0.245 (0.013)	0.202 (0.046)	0.118 (0.067)	-0.033 (0.020)	0.066 (0.032)	0.460 (0.111)	0.334 (0.056)
2015	0.362 (0.016)	0.372 (0.009)	0.263 (0.017)	0.240 (0.019)	0.173 (0.045)	0.011 (0.030)	0.039 (0.020)	0.092 (0.046)	0.324 (0.096)	0.217 (0.048)

Note: Authors' calculations using data from various editions of the Household Budget Survey. (K) indicates Kernel and (WL) Working-Leser form estimates. Standard errors in parenthesis.

Figure 1: Estimated equivalence scales for adults (LHS) and children (RHS) over time



Note: authors' calculations using data from the Household Budget Survey. Scales from Tables 1 and 2 where (KL) indicates Kernel and (WL) Working-Leser form variants of the Engel approach.

Appendix Tables A1 and A2 present estimates of child scales disaggregated by the age of child. Scales estimated using the Engel approach tend to be larger than those using other approaches, especially for younger children. The Rothbarth and demand system scales indicate a pattern of scales increasing with the age of the child, although standard errors are quite large and pairwise differences are often not statistically significant. Finally, estimates from the 3SLS specifications are sometimes large and sometimes negative, accompanied by large standard errors. The lack of precision in our estimates here is likely due to the small sample sizes that arise from looking at what can be relatively small subgroups.[16]

Taken together, three main conclusions emerge from these estimates. Firstly, while most specifications yield estimates of adult scales that are of a similar magnitude to those currently in official use (e.g. by the Irish National Scale or the modified OECD Scale), those for children are much smaller. Second, accounting for the endogeneity of total expenditure in our 3SLS specifications leads to substantially smaller estimated scales for both adults and children. Third, our estimates of scales using the AIDS and QUAIDS demand systems appear to decline over time, particularly when instruments are used to account for potential endogeneity.

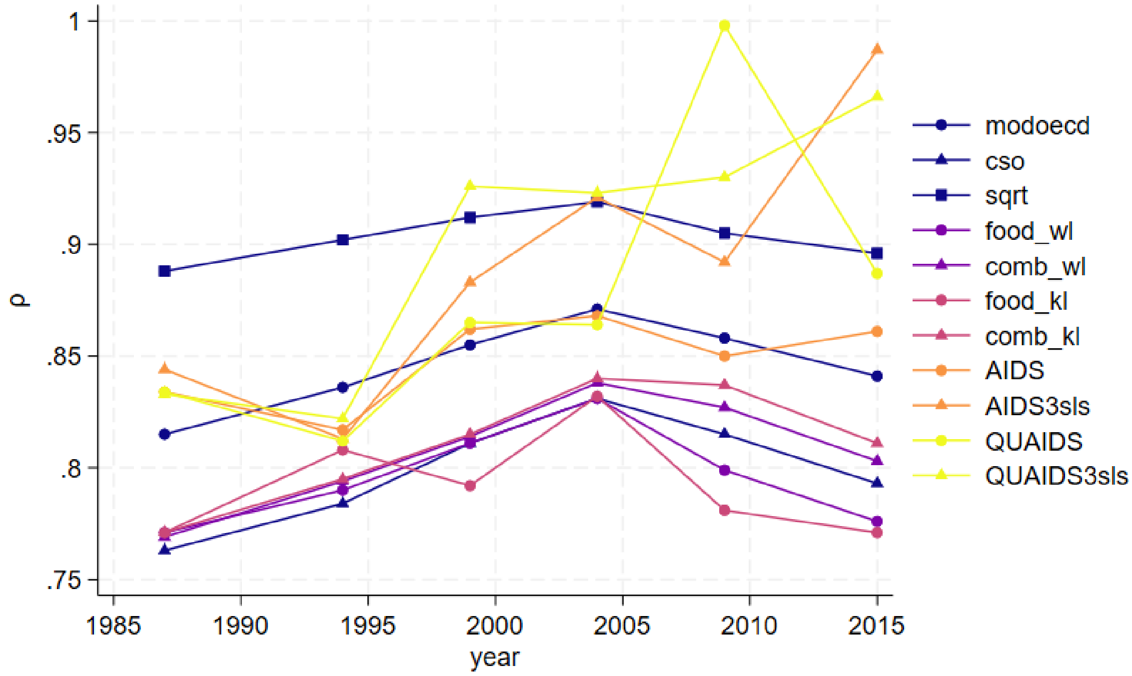
This decline is shown more clearly in Figure 1, which plots the estimated scales for adults and children from Tables 1 and 2 respectively. While estimates obtained using the Engel and (for children) Rothbarth approach are relatively constant over time, those from the demand system approaches fall – somewhat unsteadily – from 1994 onwards. The decline is particularly strong for the scales obtained using the 3SLS approach (shown with triangle markers in Figure 1).

The decline in the estimates of equivalence scales from AIDS and QUAIDS models suggests an increase in economies of scale over this period. This is also reflected in the growing correlation between households' percentile rank in the distribution of unequivalised income and their percentile rank in the distribution of income equivalised using scales derived using AIDS or QUAIDS models, as shown in Figure 2.[17] We return to the potential sources of such increasing economies of scale in our conclusion, but for now turn to look at the implications of these estimated equivalence scales for measures of poverty and inequality: one of the main uses of such scales.

[16] We also estimate Buhmann-type and two-parameter type scales in the spirit of Coulter er al. (1992), which are presented in Appendix table A.3.

[17] Appendix Figures A.1 to A.10 show the full extent of reranking using these scales.

Figure 2: Correlation in percentiles of equivalised and unequivalised income, over time



Note: authors' calculations using data from the Household Budget Survey. Scales from Tables 1 and 2 where (KL) indicates Kernel and (WL) Working-Leser form variants of the Engel approach.

4. Inequality and poverty rates

In this section we use the range of equivalence scales presented in Tables 1 and 2 to compute common measures of inequality and poverty using the HBS data described above. We also compute the same statistics using the National Scale (used by the Irish CSO), the modified OECD scale (used by EUROSTAT) and the square root scale (used by OECD) as well as without any equivalisation.

Figure 3 plots estimates of the Gini coefficient using these data. The markers are coloured by the type of scale used, with – for example – estimates based on the official ad-hoc expert scales shown in dark blue and those based on (instrumented) AIDS and QUAIDS models in yellow. Each column shows how these estimates vary across these different scales for a given year.

For most years, using official ad-hoc scales results in a higher Gini coefficient than using our estimates of equivalence scales derived from expenditure patterns. For example, in 1987 the Gini coefficient using official scales range between 0.294-0.298 (with an unequivalised Gini of 0.319) while those based on our estimated scales range from 0.285 to 0.292. An exception to this pattern is 2015, where our estimated scales result in marginally higher Gini coefficients ranging from 0.291 to 0.304: slightly above ad-hoc official scales of around 0.291.

Figure 4 shows the variation across scales in estimates of the at-risk-of-poverty (AROP) rate: the share of the population living in a household with less than 60% of median equivalised household income. In almost all of the years examined, the AROP rate based on the square root scale and the modified OECD scale is higher than those based on the Irish national scale and those derived from expenditure data. Again, an exception is 2015 when we obtain higher estimates of the AROP rate using the scales from AIDs and QUAIDS models (instrumented or otherwise).

Figure 3: Gini coefficient

Note: authors' calculations using data from the Household Budget Survey. Scales used are those presented in Tables 1 and 2 where (KL) indicates Kernel and (WL) Working-Leser form variants of the Engel approach.

Figure 4: Poverty rate (overall)

Note: authors' calculations using data from the Household Budget Survey. Scales used are those presented in Tables 1 and 2 where (KL) indicates Kernel and (WL) Working-Leser form variants of the Engel approach.

Figure 5: Poverty rate (children)

Note: authors' calculations using data from the Household Budget Survey. Scales used are those presented in Tables 1 and 2 where (KL) indicates Kernel and (WL) Working-Leser form variants of the Engel approach.

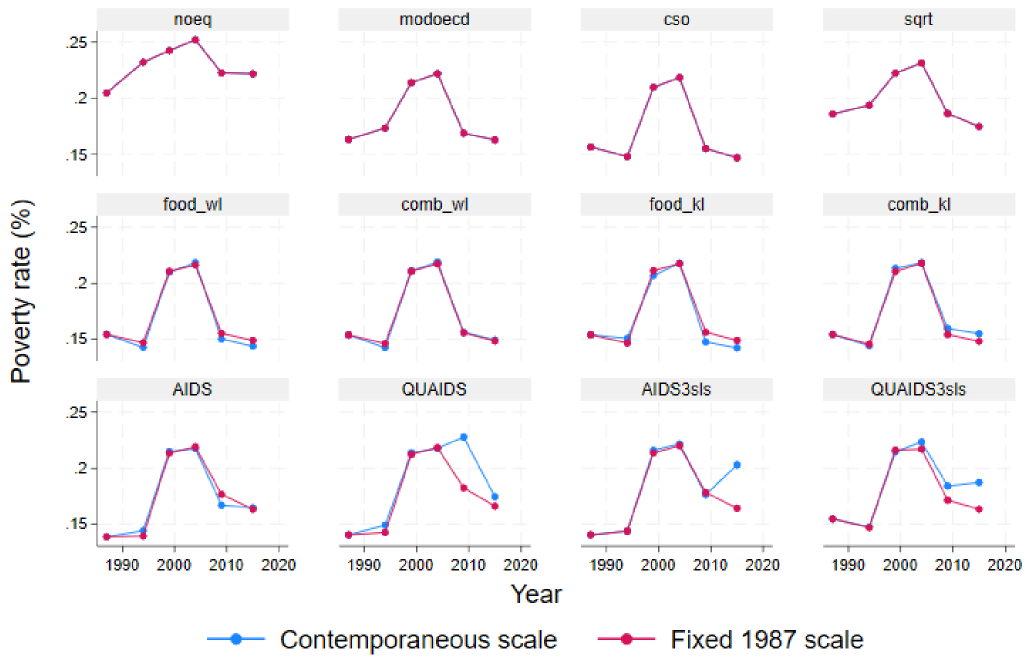
Figure 5 plots child poverty rates over time using each scale, where children are defined as aged under 14 years of age. Estimates of child poverty are considerably higher when using the official ad-hoc equivalence scales (especially the square root scale) compared with our estimated scales. For example, in 2015, the child poverty rate is 15.5 per cent when using the square root scale, 12.4 per cent when using the Irish National scale and at 10.6 per cent when using those from the (instrumented) 3SLS AIDS model.

As well as leading to different estimates of the level of poverty, the choice of scale can lead to different assessments of changes in poverty rates. So too can whether a fixed or contemporaneous equivalence scale is used. This is shown in Figures 6 and 7, which present the changes in overall (Figure 6) and child (Figure 7) poverty rates over time using the contemporaneous scales presented in Tables 1 and 2 as well as the scale estimated in 1987.

While the estimated change in the overall poverty rate is similar across the official ad-hoc and Engel scales (rising in 1999 before falling back in 2009 and declining in 2015), this differs from the pattern estimated using the demand system scales. Instead, these suggest a much smaller decline in 2009 and even – when using the 3SLS scales – a rise in 2015 when using the contemporaneous but not fixed 1987 scales.

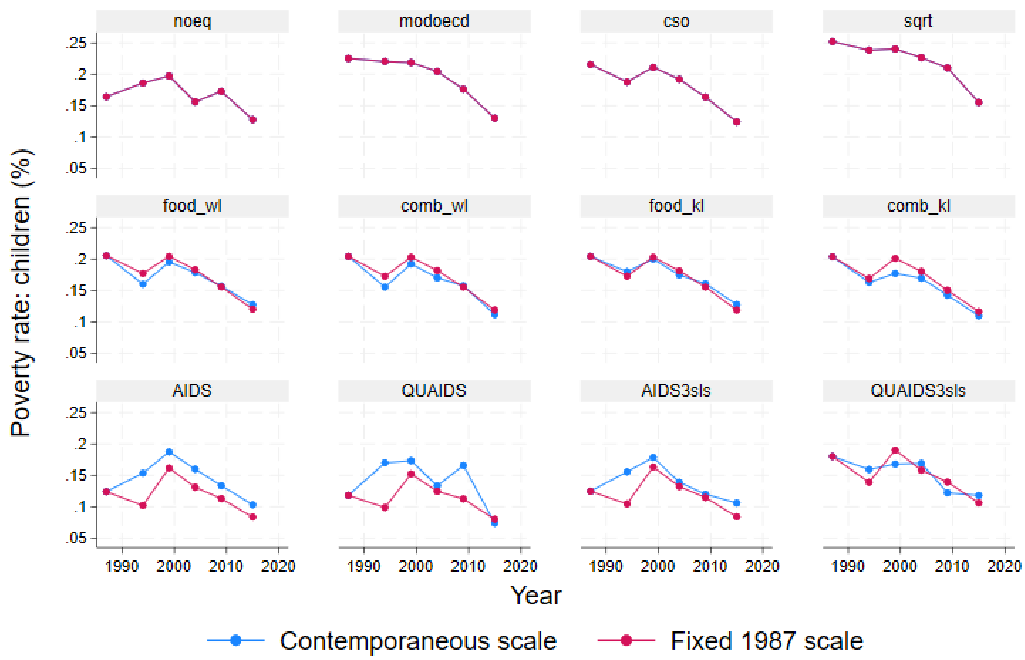
Similarly, while estimates using the official ad-hoc and Engel scales suggest child poverty rates have trended downwards over time, those estimated using contemporaneous AIDS, QUAIDS and 3SLS AIDS scales suggest a rise (from a lower level) over the 1990s which was subsequently reversed.

Figure 6: Poverty rate (overall)



Note: authors' calculations using data from the Household Budget Survey. Scales used are those presented in Tables 1 and 2 where (KL) indicates Kernel and (WL) Working-Leser form variants of the Engel approach.

Figure 7: Poverty rate (children)



Note: authors' calculations using data from the Household Budget Survey. Scales used are those presented in Tables 1 and 2 where (KL) indicates Kernel and (WL) Working-Leser form variants of the Engel approach.

5. Conclusion

This paper has examined different approaches to deriving equivalence scales using data from the Irish Household Budget Survey covering the period 1987-2015. It finds that the Engel, Rothbarth and demand system approaches all yield scales for children that are substantially smaller than those used by ad-hoc official scales like the modified OECD scale, with estimates from AIDS and QUAIDS demand systems appearing to decline over time.

Such a decline is particularly noticeable when a 3SLS estimator is used to correct for the potential endogeneity of total expenditure, which also leads to substantially smaller estimated scales for both adults and children. These estimates suggest an increase in economies of scale over the period we examine, something which is reflected in the growing correlation between households' percentile rank in the distribution of unequivalised income and income equivalised using scales derived from AIDS or QUAIDS demand systems.

A potential source of such increasing economies of scale is the sharp rise in housing costs that has been experienced in Ireland over recent decades. Keely and Lyons (2020) showed rents have risen by 166% and sales prices by 384% between 1987 and 2018 in real terms, while our HBS data indicate that housing has doubled as a share of total expenditure between 1987 and 2015. Given that housing expenditure – the canonical public good of Deaton and Paxson (1998) – embodies substantial economies of scale, a shift in expenditure towards housing might therefore be expected to increase household economies of scale in total expenditure.

Changing economies of scale in household expenditure of this kind will be at most only partially captured in estimates of equivalence scales derived using the Engel or Rothbarth approaches. This suggests additional grounds to favour equivalence scales estimated using demand system approaches over and above their greater consistency with consumer theory (Deaton and Muellbauer, 1986) and the impossibility of estimating adult scales with the Rothbarth approach. Scales derived from AIDS and QUAIDS models that can incorporate information on households' wider expenditure and prices should therefore be more widely used, especially over the longer-run when expenditure patterns and prices are changing.

Our results also have implications for those engaged in the estimation of poverty and inequality statistics using ad-hoc official scales. At a minimum, it appears sensible to test the sensitivity of results – particularly poverty indices – to the use of alternative equivalence scales. It may also – especially when considering longer-run changes – be important to consider the effects of using contemporaneous rather than fixed scales. Above all, researchers should provide explicit justification for their choice of equivalence scale given the first-order difference that the choice of scale can have for estimates of poverty in particular.

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Appendix

Table A1: Estimated Equivalence Scales for Children by Age, Single-Equation Models

		1987	1994	1999	2004	2009	2015
Food (WL)	0-4	0.558 (0.004)	0.566 (0.55)	0.568 (0.005)	0.560 (0.005)	0.546 (0.006)	0.534 (0.006)
	5-13	0.206 (0.013)	0.182 (0.016)	0.173 (0.018)	0.207 (0.022)	0.232 (0.033)	0.286 (0.035)
	14-17	0.361 (0.024)	0.338 (0.028)	0.325 (0.027)	0.304 (0.030)	0.413 (0.045)	0.394 (0.050)
Food (K)	0-4	0.545 (0.003)	0.541 (0.003)	0.557 (0.008	0.547 (0.002)	0.532 (0.004)	0.523 (0.005)
	5-13	0.259 (0.007)	0.278 (0.014)	0.232 (0.005)	0.253 (0.008)	0.279 (0.030)	0.332 (0.013)
	14-17	0.392 (0.016)	0.377 (0.024)	0.235 (0.005)	0.351 (0.013)	0.470 (0.038)	0.422 (0.009)
Combined (WL)	0-4	0.566 (0.006)	0.566 (0.007)	0.568 (0.007)	0.576 (0.008)	0.552 (0.013)	0.547 (0.011)
	5-13	0.201 (0.018)	0.178 (0.023)	0.186 (0.027)	0.246 (0.032)	0.322 (0.044)	0.267 (0.040)
	14-17	0.371 (0.030)	0.310 (0.038)	0.210 (0.032)	0.136 (0.030)	0.158 (0.035)	0.089 (0.030)
Combined (K)	0-4	0.550 (0.000)	0.549 (0.004)	0.556 (0.004)	0.559 (0.002)	0.532 (0.000)	0.538 (0.005)
	5-13	0.252 (0.009)	0.257 (0.020)	0.231 (0.011)	0.296 (0.039)	0.315 (0.023)	0.293 (0.035)
	14-17	0.393 (0.011)	0.324 (0.041)	0.234 (0.024)	0.166 (0.030)	0.123 (0.009)	0.111 (0.041)
Rothbarth (Alcohol)	0-4	0.159 (0.054)	0.174 (0.058)	0.067 (0.045)	0.116 (0.061)	0.149 (0.089)	0.200 (0.108)
	5-13	0.091 (0.029)	0.103 (0.032)	0.134 (0.036)	0.135 (0.069)	0.175 (0.055)	0.162 (0.057)
	14-17	0.124 (0.089)	0.157 (0.056)	0.169 (0.056)	0.212 (0.067)	0.443 (0.121)	0.287 (0.110)
Rothbarth (Combined)	0-4	0.031 (0.026)	0.038 (0.028)	0.011 (0.027)	0.025 (0.033)	-0.003 (0.038)	-0.048 (0.035)
	5-13	0.130 (0.023)	0.139 (0.025)	0.137 (0.027)	0.158 (0.029)	0.180 (0.037)	0.126 (0.035)
	14-17	0.157 (0.056)	0.202 (0.045)	0.211 (0.045)	0.197) (0.045)	0.317 (0.063)	0.252 (0.068)

Table A2: Estimated Equivalence Scales for Children by Age, Demand System Models

		1987	1994	1999	2004	2009	2015
AIDS	0-4	-0.023 (0.033)	0.062 (0.073)	0.157 (0.047)	0.206 (0.061)	0.212 (0.095)	0.149 (0.088)
	5-13	0.201 (0.032)	0.372 (0.065)	0.228 (0.035)	0.250 (0.043)	0.223 (0.067)	0.211 (0.063)
	14-17	0.384 (0.068)	0.426 (0.108)	0.407 (0.063)	0.377 (0.078)	0.424 (0.127)	0.301 (0.106)
QUAIDS	0-4	-0.052 (0.032)	0.052 (0.060)	0.065 (0.048)	0.028 (0.056)	0.005 (0.044)	-0.012 (0.082)
	5-13	0.180 (0.031)	0.390 (0.050)	0.205 (0.040)	0.244 (0.051)	-0.066 (0.022)	0.063 (0.055)
	14-17	0.352 (0.066)	0.470 (0.090)	0.417 (0.074)	0.418 (0.099)	0.046 (0.050)	0.234 (0.103)
AIDS (3SLS)	0-4	-0.075 (0.051)	0.135 (0.088)	-0.007 (0.074)	-0.115 (0.065)	-0.086 (0.092)	0.055 (0.055)
	5-13	0.268 (0.058)	0.384 (0.073)	0.263 (0.069)	0.203 (0.076)	0.310 (0.098)	-0.036 (0.024)
	14-17	-0.162 (0.117)	-0.248 (0.127)	0.043 (0.139)	-0.035 (0.167)	-0.119 (0.202)	0.041 (0.055)
QUAIDS (3SLS)	0-4	0.008 (0.053)	0.059 (0.067)	-0.053 (0.048)	0.157 (0.066)	-0.070 (0.061)	0.183 (0.104)
	5-13	0.359 (0.048)	0.368 (0.055)	0.188 (0.039)	0.164 (0.038)	0.156 (0.048)	0.038 (0.047)
	14-17	-0.335 (0.087)	-0.131 (0.110)	-0.100 (0.077)	-0.136 (0.082)	-0.102 (0.106)	-0.259 (0.082)

Table A.3 Buhmann-type and two-parameter-type scales

	1987	1994	1999	2004	2009	2015
Buhmann-type	0.705 (0.010)	0.813 (0.010)	0.842 (0.012)	0.846 (0.014)	0.680 (0.013)	0.749 (0.012)
Two-parameter-type						
ε	0.916 (0.013)	0.977 (0.013)	0.982 (0.014)	0.963 (0.015)	0.850 (0.016)	0.869 (0.015)
η	0.299 (0.017)	0.385 (0.020)	0.463 (0.023)	0.552 (0.027)	0.351 (0.025)	0.493 (0.029)

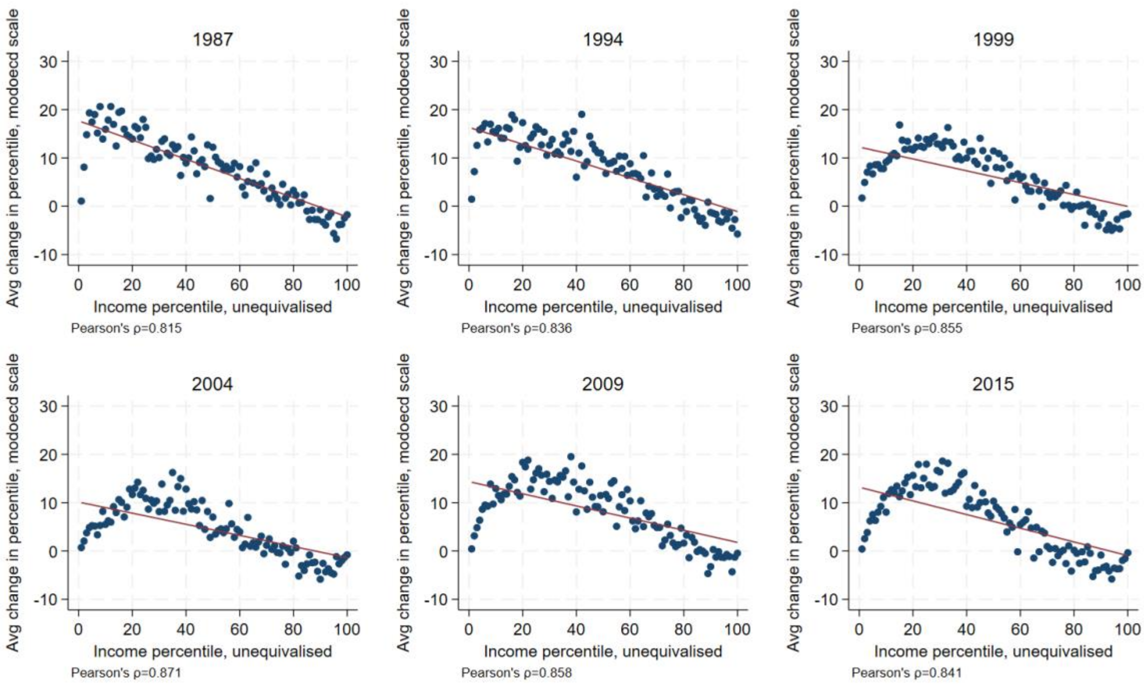
Note: Own calculations using HBS. Parameter names follow (Jenkins and Cowell 1994). Standard errors in parentheses.

Table A4: Estimates of the Gini Coefficient and Overall Poverty Rate by scale over time

	1987	1994	1999	2004	2009	2015
A. Gini						
No equiv	0.319	0.311	0.324	0.349	0.340	0.318
Modified OECD	0.294	0.294	0.298	0.317	0.316	0.290
CSO	0.294	0.296	0.298	0.317	0.316	0.291
Square Root	0.298	0.294	0.301	0.322	0.319	0.291
Food (WL)	0.292	0.295	0.297	0.316	0.316	0.292
Combined (WL)	0.292	0.294	0.296	0.316	0.315	0.291
Food (K)	0.292	0.294	0.298	0.316	0.317	0.292
Combined (K)	0.291	0.295	0.296	0.316	0.313	0.292
AIDS	0.285	0.293	0.295	0.316	0.313	0.291
AIDS 3SLS	0.285	0.293	0.294	0.319	0.312	0.304
QUAIDS	0.285	0.294	0.295	0.317	0.341	0.299
QUAIDS 3SLS	0.287	0.293	0.295	0.318	0.313	0.296
B. Poverty rate (overall)						
No equiv	20.5%	23.2%	24.2%	25.2%	22.3%	22.2%
Modified OECD	16.3%	17.3%	21.4%	22.2%	16.9%	16.3%
CSO	15.7%	14.8%	21.0%	21.8%	15.5%	14.7%
Square Root	18.6%	19.4%	22.2%	23.1%	18.6%	17.5%
Food (WL)	15.4%	14.3%	21.0%	21.8%	15.0%	14.4%
Combined (WL)	15.4%	14.3%	21.1%	21.9%	15.6%	14.9%
Food (K)	15.4%	15.1%	20.7%	21.8%	14.8%	14.2%
Combined (K)	15.4%	14.4%	21.3%	21.8%	16.0%	15.5%
AIDS	13.9%	14.4%	21.5%	21.7%	16.7%	16.5%
AIDS 3SLS	14.1%	14.4%	21.6%	22.2%	17.6%	20.3%
QUAIDS	14.1%	14.9%	21.4%	21.8%	22.8%	17.4%
QUAIDS 3SLS	15.5%	14.8%	21.5%	22.3%	18.4%	18.7%
C. Poverty rate (children)						
No equiv	16.4%	18.6%	19.7%	15.6%	17.3%	12.8%
Modified OECD	22.5%	22.1%	21.9%	20.5%	17.6%	13.0%
CSO	21.6%	18.8%	21.1%	19.2%	16.4%	12.4%
Square Root	25.2%	23.8%	24.0%	22.7%	21.0%	15.5%
Food (WL)	20.6%	16.0%	19.5%	17.9%	15.7%	12.7%
Combined (WL)	20.4%	15.5%	19.2%	17.0%	15.8%	11.2%
Food (K)	20.4%	18.0%	20.0%	17.5%	16.1%	12.8%
Combined (K)	20.4%	16.3%	17.7%	17.0%	14.2%	11.0%
AIDS	12.4%	15.3%	18.7%	16.0%	13.4%	10.3%
AIDS 3SLS	12.5%	15.6%	17.9%	13.9%	12.0%	10.6%
QUAIDS	11.8%	17.0%	17.3%	13.3%	16.6%	7.4%
QUAIDS 3SLS	18.0%	15.9%	16.8%	16.9%	12.2%	11.8%

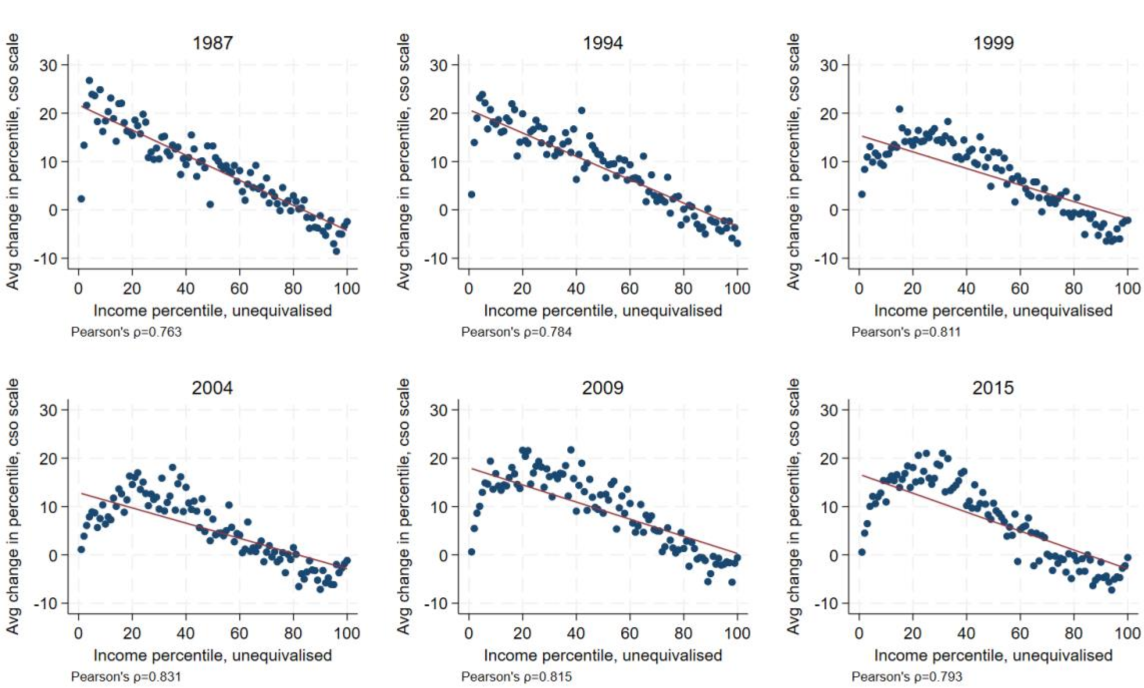
Note: authors' calculations using data from the Household Budget Survey. Scales used are those presented in Tables 1 and 2 where (KL) indicates Kernel and (WL) Working-Leser form variants of the Engel approach.

Figure A.1 Comparison of percentile rank in unequivalised and equivalised income: modified OECD scale



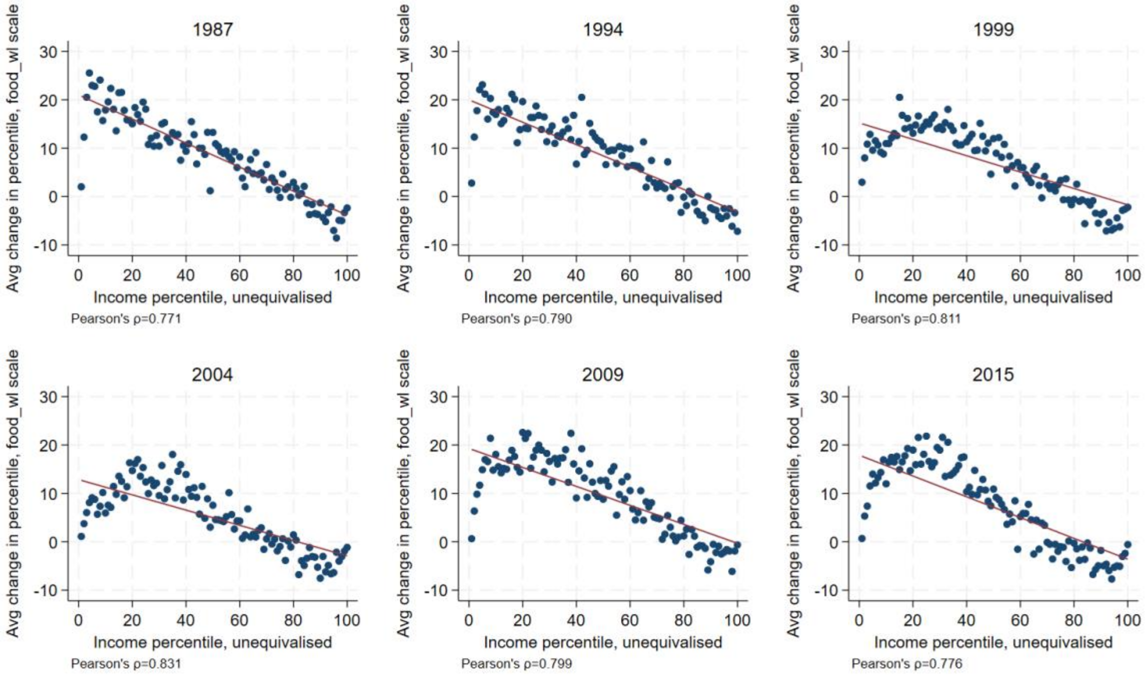
Note: authors' calculations using data from the Household Budget Survey. Scales used are those presented in Tables 1 and 2

Figure A.2 Comparison of percentile rank in unequivalised and equivalised income: CSO national scale



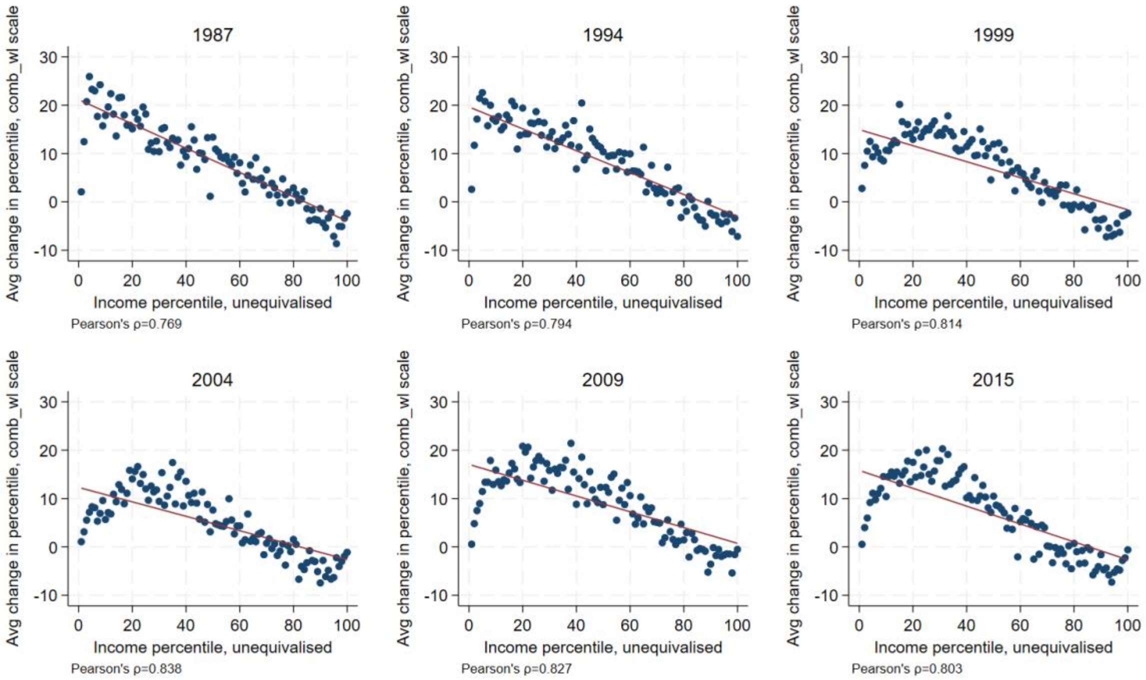
Note: authors' calculations using data from the Household Budget Survey. Scales used are those presented in Tables 1 and 2

Figure A.3 Comparison of percentile rank in unequivalised and equivalised income: Food (WL) scale



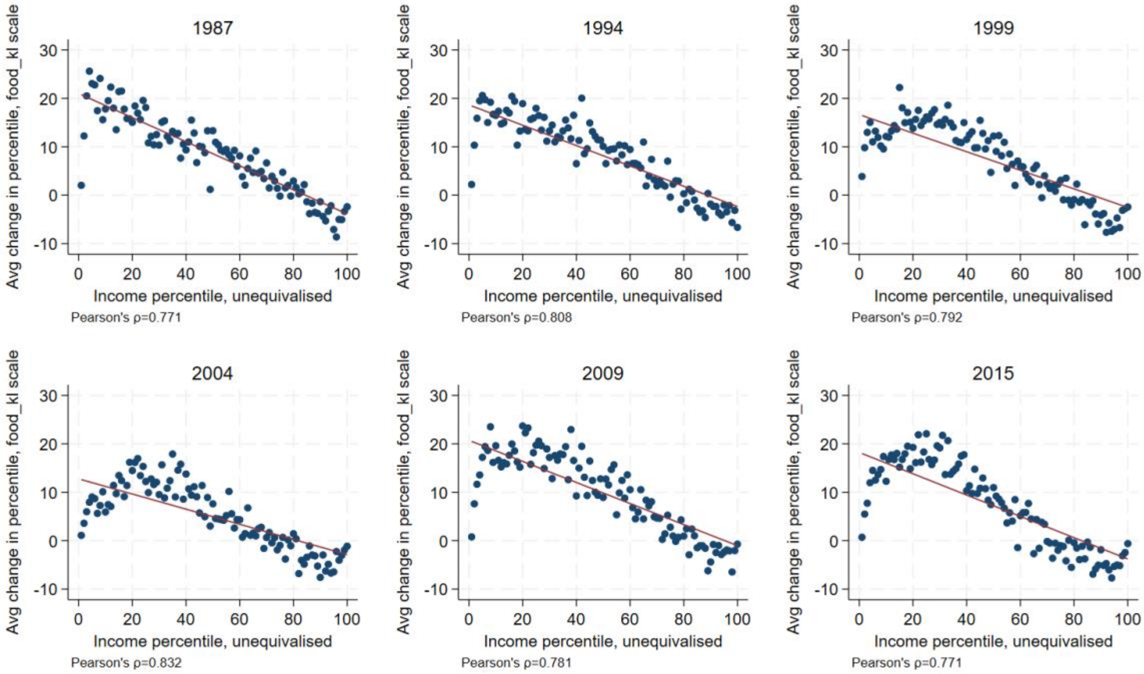
Note: authors' calculations using data from the Household Budget Survey. Scales used are those presented in Tables 1 and 2

Figure A.4 Comparison of percentile rank in unequivalised and equivalised income: Combined (WL) scale



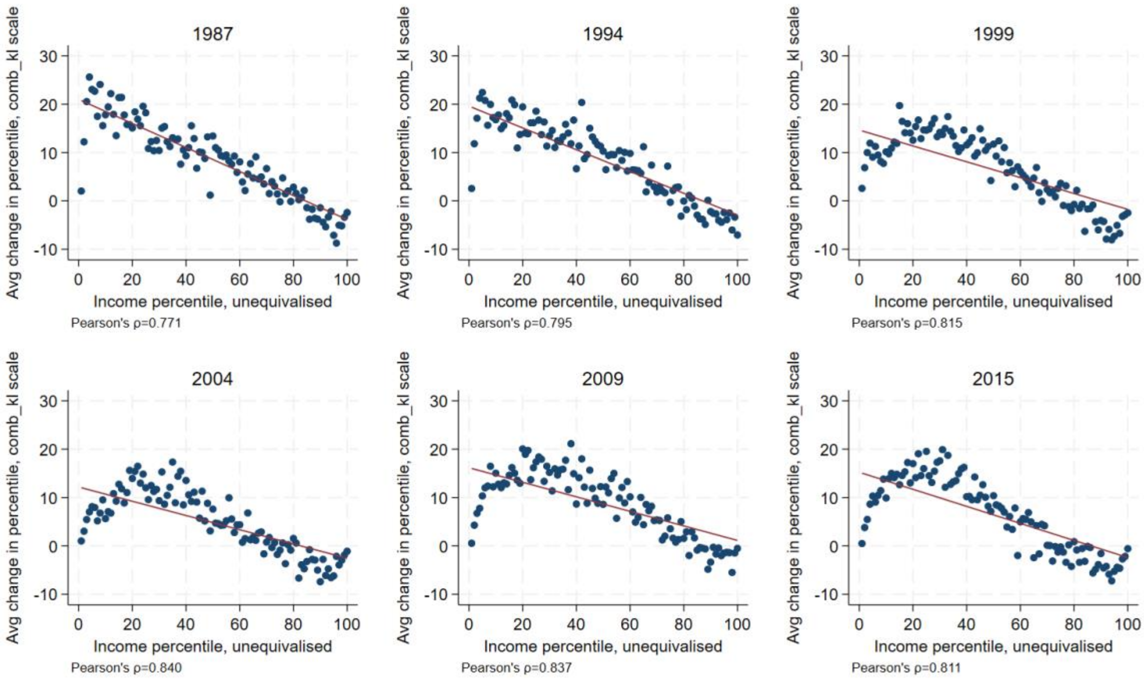
Note: authors' calculations using data from the Household Budget Survey. Scales used are those presented in Tables 1 and 2

Figure A.5 Comparison of percentile rank in unequivalised and equivalised income: Food (K) scale



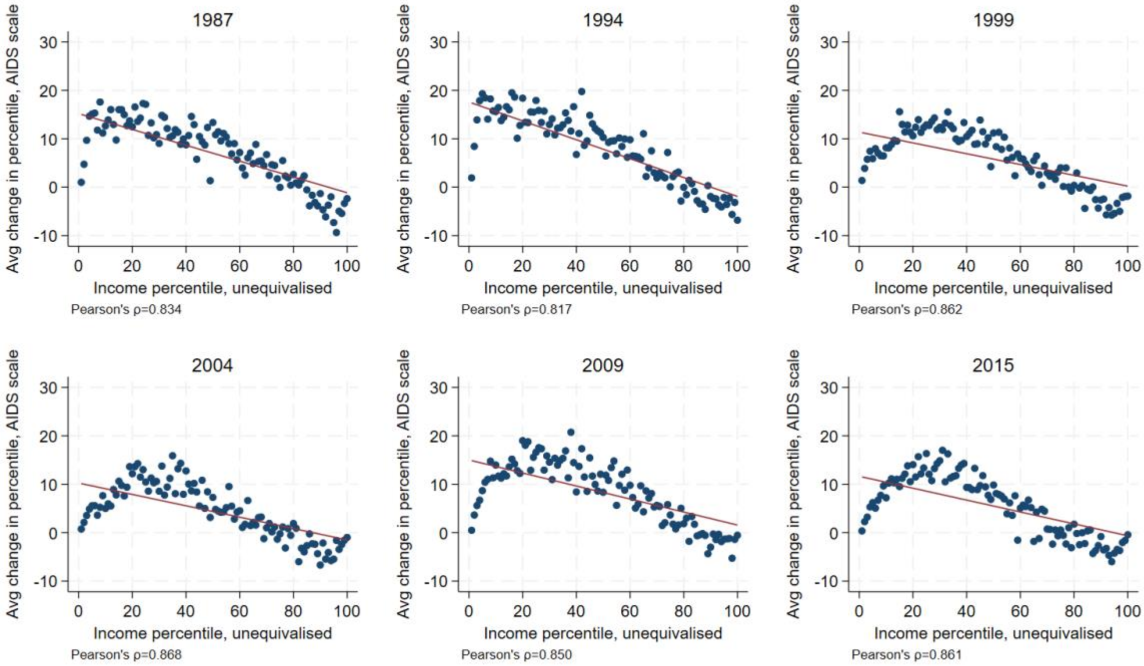
Note: authors' calculations using data from the Household Budget Survey. Scales used are those presented in Tables 1 and 2

Figure A.6 Comparison of percentile rank in unequivalised and equivalised income: Combined (K) scale



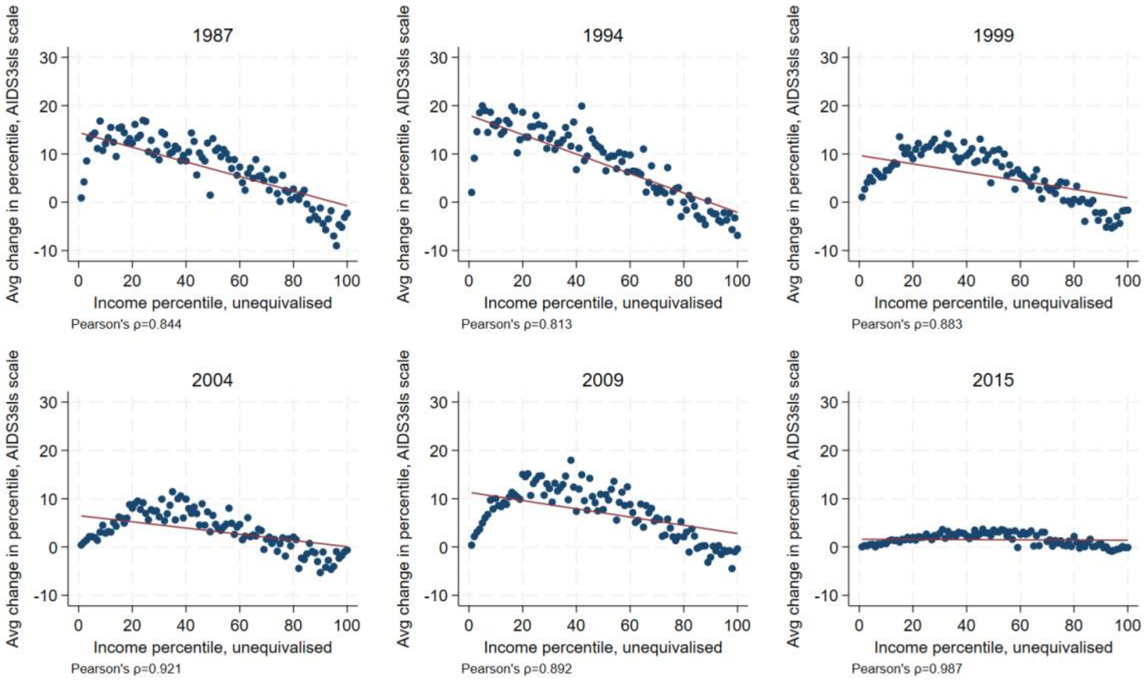
Note: authors' calculations using data from the Household Budget Survey. Scales used are those presented in Tables 1 and 2

Figure A.7 Comparison of percentile rank in unequivalised and equivalised income: AIDS scale



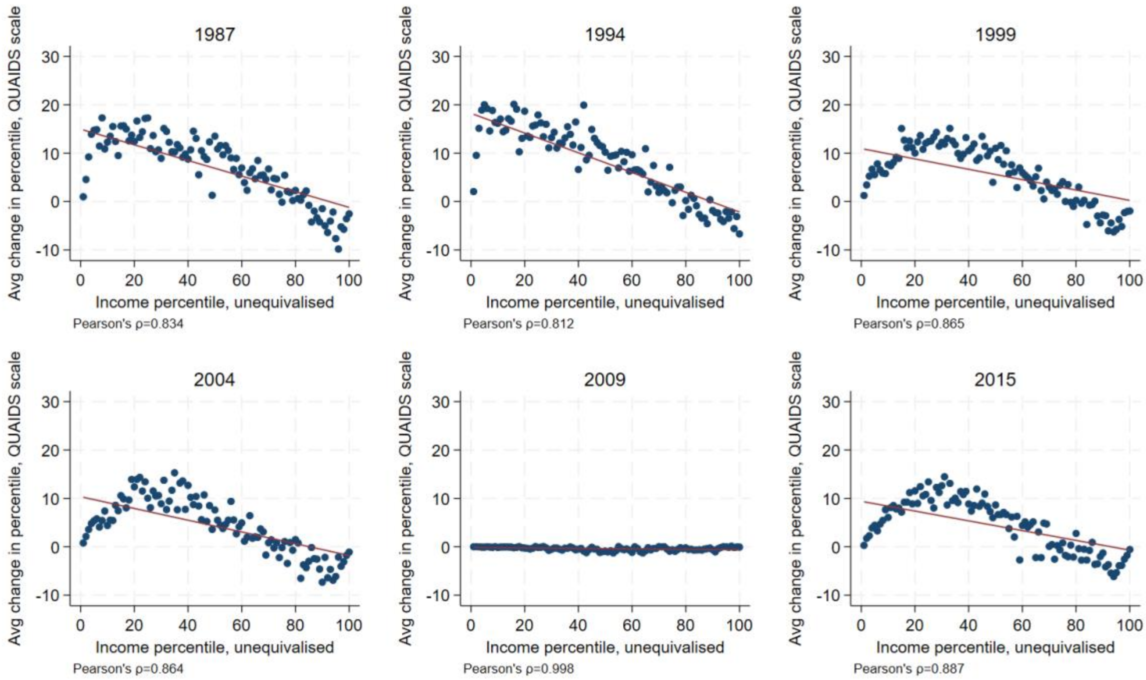
Note: authors' calculations using data from the Household Budget Survey. Scales used are those presented in Tables 1 and 2

Figure A.8 Comparison of percentile rank in unequivalised and equivalised income: AIDS 3SLS scale



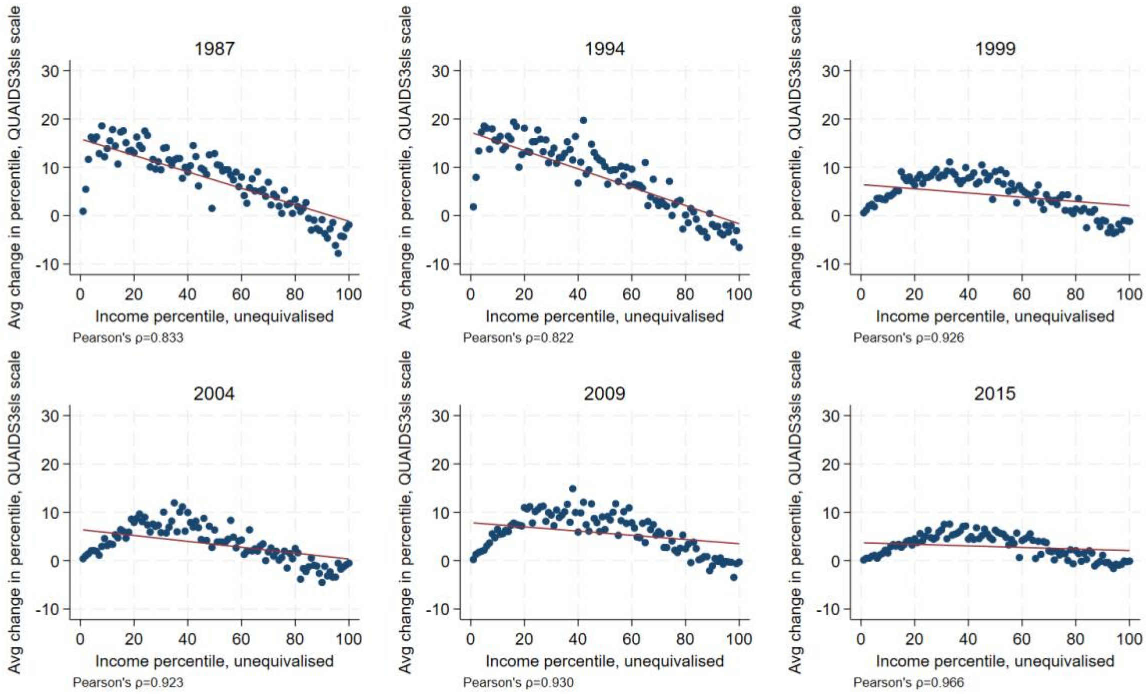
Note: authors' calculations using data from the Household Budget Survey. Scales used are those presented in Tables 1 and 2

Figure A.9 Comparison of percentile rank in unequivalised and equivalised income: QUAIDS scale



Note: authors' calculations using data from the Household Budget Survey. Scales used are those presented in Tables 1 and 2

Figure A.10 Comparison of percentile rank in unequivalised and equivalised income: QUAIDS 3SLS scale



Note: authors' calculations using data from the Household Budget Survey. Scales used are those presented in Tables 1 and 2